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FEATURES

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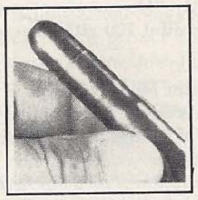

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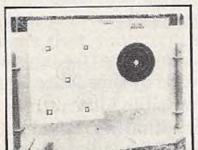
DEPARTMENTS



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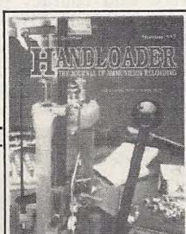


On the cover . . . The Dillon Precision 550 progressive reloader, set up here for the .45 Colt with Hornady XTP bullets and Winchester Large Pistol primers, can handle a broad range of calibers, from the smallest .22 centerfire to the .45-70. Photo by Gerald Hudson

Groups, Statistics

G. Sitton

THIS rifle is more than sufficiently accurate. With selected loads, it will put three shots into an inch at 100 yards whenever I do my part. And three-shot groups are all I need to know about in a big game rifle. Deer and such rarely hang around long enough to make five or 10-shot accuracy an issue.

How many times have you read assertions of this sort in the shooting press? Truth be told, I've written similar lines myself, though I knew in an instinctive way that the underlying assumptions were flawed. What do the words mean, if anything, in terms of actual fact?

This latter question came up during a visit with Editor Dave Scovill in Prescott last February. Both of us knew reporting test results based on three-shot groups provokes heated correspondence from some readers. Neither of us knew exactly why such groups are deemed inadequate for the evaluation of rifles, handguns and ammunition for same.

We agreed that it would be interesting to examine the relative merits (if any) of three-shot groups versus the more respectable strings of five and 10 rounds. We also agreed that there are often rather compelling reasons of a pragmatic nature which favor the three shot group.

The point of shooting groups, regardless of the number of shots included, is simple enough once ego and pure pleasure are put aside. We want to determine the *typical* accuracy that can be expected from a particular lot of ammunition in a specific firearm. Whether game will, or will not, stand for more than three shots is irrelevant if three-shot groups cannot be used to produce credible predictions of the system's future performance.

Obviously it is important that this determination be made with a high degree of reliability. Holding the required expenditure of time, ammunition and effort to a minimum is also desirable in most cases. I did not know at the outset of this project what a series of three-shot groups actually meant as a predictor of accuracy.

If the results are to be significant in any comparative way, they must be expressed in a statistically sound, generally comprehensible form. Failing that, our tests create only isolated facts, incapable of meaningful relation to other information and experience. Data of this kind are rarely worth having.

I wanted to know if the three-shot group could be made to satisfy these criteria and, if not, why not.

My quest began with a telephone call to John Wootters. John has long been a strong and faithful advocate of the five-shot group. He formerly insisted that five groups of five shots each were needed to dependably test a given load. Latterly, he has reduced the number of groups required to four.

John told me he was convinced that, when testing two loads in order to select the more accurate one, firing 20 rounds of each ammunition into four groups of five (or two groups of 10) will provide a satisfactory degree of probability that the correct choice is being made. Firing more than 20 rounds increases this degree of probability, but incremental gains are very small in relation to ammunition expended.

Of course, when the comparative results are very close, it may be necessary to repeat the test. However, the conclusion reached after another 20 rounds of each load are fired may still be no more definitive than that the loads are quite similar in accuracy. When this occurs, selection of one load over the other will likely be based on characteristics other than accuracy — velocity, bullet construction and ballistic coefficient come to mind.

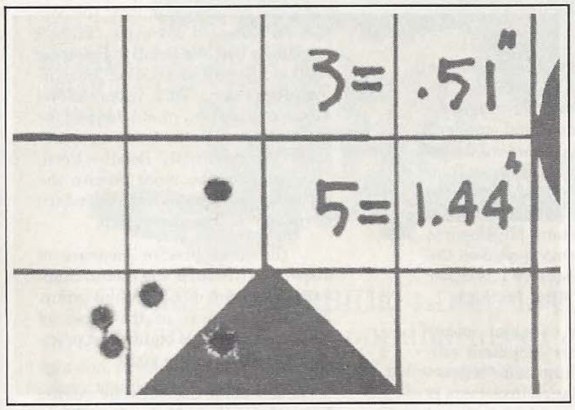
John referred me to an article by William C. Davis, Jr., entitled "Determining Rifle Accuracy" (*Handloading*, NRA 1986). Mr. Davis has an uncommon gift for making complex statistical matters accessible. Without poaching too heavily on his writings, the salient points made are:

- Assuming an equal number of rounds are expended, the information yield from five-shot groups is about as

The Model 7 Remington .243 Winchester is typical of sporting rifles that may be appropriately tested with three-shot groups.

reliable as that derived from groups of 10 shots.

- Given the average of any number of five-shot groups from one to 100, there are known limits within which the population mean, or long-run average, will probably lie. "Probably" in this case is defined as nine chances in 10, or 90 percent.
- When 20 rounds are fired in four groups of five shots, there is a 90 percent probability that the average group size established by firing a very large number of groups (long-run) will fall within precise limits. These limits are calculated by multiplying the test average group size by .77 and 1.23. Thus, if the average of four five-shot groups is 1.13 inches, the long-run average will probably fall between .87 and 1.40 inches.



The first three shots from the Model 7 cluster nicely into .51 inch; two more open the group to mediocre proportions.

and Practical Facts

In the great majority of accuracy tests, four groups of five shots will serve as an adequate basis for evaluation and judgment. It seems, too, that the information derived from any smaller sample would be so imprecise as to be practically useless.

To substantially refine the predictions made possible by firing 20 rounds (4x5), a gross amount of shooting is required. We can reduce the range within which the long-run average is likely to lie, of course. Shoot 100 rounds in five-shot strings and the low-end factor rises to .9, while the high-end factor drops to 1.1. The relative efficiency with which this increased precision is gained is, for most purposes, not particularly attractive.

The matter was discussed with Dr. Ken Oehler of chronograph fame; Bob Forker, a professional ballistician and handloading editor for *Guns & Ammo*; and Kevin Thomas in Sierra's ballistics lab. All were informative and helpful in filling out the brief for five-shot groups. Some points of interest made by these gentlemen follow:

- We know what five-shot groups mean in terms of widely published and statistically valid data. Five-shot groups give meaning to the MOA measurement by which we judge accuracy and this measurement is meaningless if the shot count is not constant.
- Firing only three shots lessens the likelihood of including one of the very bad shots present in a normal ammunition population in any given group. By the same token, a truly bad three-shot group does favor the conclusion that the ammo is bad. The converse conclusion cannot be reached on the basis of a good three-shot group.

Mr. Forker, in a moment of prophetic vision, opined that two or three times the number of five-shot groups would probably be needed to reach the same level of certainty with three-shot groups.

At this point, I knew from several different sources what was right about five-shot groups. I still didn't know exactly what was wrong with three-shot groups. Their most glaring deficiency, so far as I had learned, was that they lacked two shots to make five.

The problem was galling. Call it skepticism, an independent mind, or natural rebelliousness, I have trouble accepting anything just because "that's the way it is." Still, frustration mounted. "Nobody knows" is a lame conclusion.

Then, I spoke to Ken Kees at Speer. Actually, he is division quality manager for the Sporting Equipment Division of Blount. More important than his title, Ken was interested in the questions I posed. He asked me to call him again in a few days. Meanwhile, he would send me a copy of a monograph by one Frank E. Grubbs, Ph.D. on "Statistical Measures of Accuracy for Riflemen and Missile Engineers" (Copyrighted 1964). Dr. Grubbs was/is a clever, agile statistician. Alas, his monograph I found heavy going.

A few days passed. When I called again, Ken and his associates had some very useful, not to mention surprising, answers. Following is a slightly abridged version of a memo summarizing their findings:

> This memo is hopefully the answer to your question regarding the adequacy of three shot versus five shot groups to assess accuracy in the most efficient (fewest rounds fired) manner. The monograph by Dr. Frank E. Grubbs . . . was used as an initial reference and our resident statistician and quality engineer, Dr. Brent Banister, finished the analysis as presented here.
>
> What we want to define is a number that relates to group size rather than how close we are to the "bullseye" . . . We set out to find the optimum sample size needed to best

estimate the true dispersion of a load with a minimum number of rounds fired.

Dr. Grubbs measures the dispersion of a group in a variety of ways, including mean radius (average individual dispersion), radial standard deviation (combined standard deviation of x and y components away from the center of impact) and extreme spread (distance between farthest holes in any direction). Most people decide to use extreme spread as the preferred measure because it is easier to calculate than other methods.

The reason to consider other methods is . . . their increased efficiency in determining "true" dispersion. True or population dispersion is the pattern that would be expected if a large number of rounds of the same lot were fired . . . . True dispersion is defined here as the population standard deviation.

A single group measure can be converted to an estimate of true dispersion or population standard deviation by a correction factor which depends on the number of rounds in the group and the type of measurement method used. These factors are listed by Grubbs as the Mean Value for each method of measuring dispersion. The Standard Deviation that is listed refers to the limits of confidence in the Mean Value.

The coefficient of variation is a measure of Relative Precision determined by dividing the Standard Deviation by the Mean Value. This gives a comparison of relative sampling error when estimating the population standard deviation. Table 9 by Grubbs lists the Relative Precision for the different methods of measuring dispersion. This value allows direct comparison of efficiencies for the different methods of measurement. The smaller the Relative Precision number, the more precise the estimate of population standard deviation or true dispersion.

. . . the most precise measure of dispersion [listed] is the Radial Standard Deviation of a 20-round group. Notice that most of the types of measurement have equivalent precision for small group sizes.

The key point that must be carried forward is the effect of multiple groups on Relative Precision, especially if it is desired to minimize the total number of rounds fired. By using the average of several group measures, the precision of the estimate of true dispersion is increased by the factor $\sqrt{g}$, the square root of the number of groups tested. The Relative Precision value in Table 9 is corrected by dividing by $\sqrt{g}$. We will define this new value as Relative Group Precision.

In order to determine the best combination of group size and number of groups tested to maximize the precision of the estimate of true dispersion, the value g is broken down into its factors:

$$g = N/n$$

where N = total rounds fired

n = sample size (number of rounds in group)

If the total number of rounds fired is assumed the same for two different groups (i.e., 3x5-shot groups versus 5x3-shot groups = 15 rounds), then a direct comparison can be made for each group size using a value we will define as Group Efficiency (GE), which is equivalent to:

$$GE = RP^2 * (n/N)$$

If N = 1 (for convenience), RP = Relative Precision and n = group size, then:

$$GE = RP^2 * n$$

Using the measure of Extreme Spread, if Group Efficiency is plotted for each sample size (see graph), the optimum Group Efficiency is found for groups of size 7, although groups of 5 and 6 are very close. Grubbs admitted to possible rounding error in the third decimal when he generated the Relative Precision values for Extreme Spread, which may cause the lack of smoothness in the curve.

This means that the most efficient method of determining precision using the measure of Extreme Spread is to shoot groups of 7 rounds.

The Group Efficiency for 3-, 5-, 7- and 9-shot groups is:

$$GE (3) = (.371)^2 * 3 = .413$$
$$GE (5) = (.269)^2 * 5 = .361$$
$$GE (7) = (.225)^2 * 7 = .355$$
$$GE (9) = (.202)^2 * 9 = .367$$

The Relative Group Efficiency is then equal to the ratios of the Group Efficiencies or for the 3- versus 5-shot groups:

$$RGE = (.361/.413) = 87\%$$

This means that five-shot groups are 13 percent more efficient than three-shot groups in estimating the true dispersion for the same number of rounds fired.

The problem can be worked in reverse by equating the Relative Group Precision for each group and solving for g (number of groups), given one group total for comparison.

The number of three-shot groups required to give the same precision as four five-shot groups is found by solving:

$$RP_5/\sqrt{g_5} = RP_3/\sqrt{g_3}$$
$$.269/\sqrt{4} = .371/\sqrt{g_3}$$

g = 7.6 groups or 22.8 (23) rounds where 23 is about 13 percent more rounds than 20.

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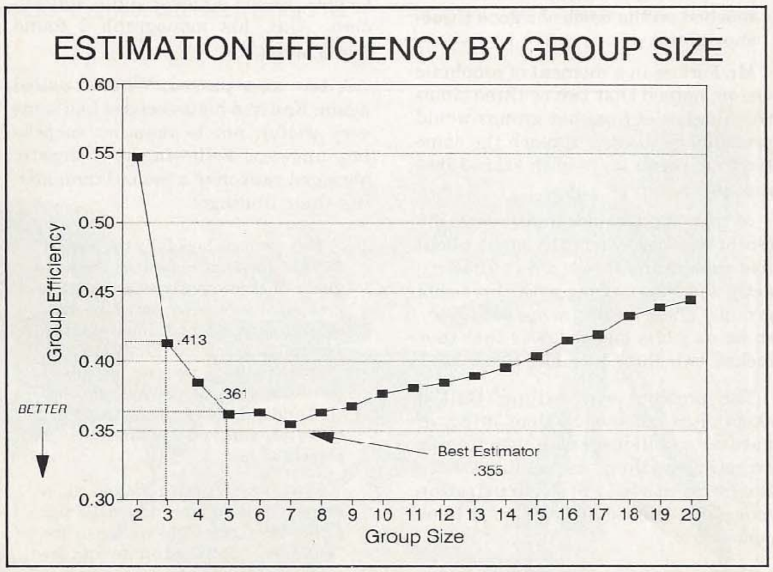

ESTIMATION EFFICIENCY BY GROUP SIZE

Groups, Statistics & Practical Facts

(continued from page 44)

Similar comparisons can be made for the other measures of dispersion as listed by Grubbs.

At last, quantified, factual reality was brought to bear on the three-shot group. They actually have statistically reliable meaning! We can now calculate quite easily the number of three-shot groups needed to equal the precision of any given number of five-shot groups. All we have to do is solve for g as above or multiply the number of rounds fired into five-shot groups by 1.13; then, round up to next whole number divisible by 3.

If we want to equal the quality of data derived from firing 20 groups of five shots (100 rounds), we must fire 114 rounds into 38 three-shot groups. Note here how close Bob Forker came with his guesstimate. Approximately twice the number of five-shot groups are required to produce data of comparable reliability with threes.

Mr. Kees and Dr. Banister have assured me that we can use Mr. Davis' factoring scale to compute the probable limits of long-run averages by substituting the required number of three-shot groups for the appropriate number of five-shot strings. There will be a small error introduced when rounding up to the next complete three-shot group, but it works for, not against, the reliability of the resulting estimate.

The emergence of the seven shot group as the most efficient means of assessing accuracy when Extreme Spread is the measure may be no more than a curiosity. The makers of ammunition and components are not likely to abandon packages of 20, 50 and 100 units for new ones containing 21, 49 and 98. The bench shooters presumably won't rush to increase their courses of fire to seven and 14 rounds, either. The superiority of the seven-shot group may, however, relate in some way to the federal government's insistence on 14-shot groups when procuring 168-grain MatchKing .308s from Sierra; maybe not.

Despite the relative inefficiency of three-shot groups, as opposed to fives, factors unrelated to the firearms and ammunition frequently mitigate in favor of the shorter strings. Furthermore, some rifles will shoot very nice three-shot groups, but simply will not deliver accuracy equal to their intended purpose when two more shots are added to the string, no matter what the load. This assumes that groups are fired quickly enough to heat the barrel.

A recently acquired Remington Model Seven FS in .243 Winchester is meant for called predators, running jackrabbits, javelina and general duty in southern Arizona's brushy, broken terrain. With Redfield two-piece bases and rings, a Burris Mini 3-9x scope and a nylon carrying strap, it weighs but 6 pounds, 11 ounces. That's just right for hard walking through country where a 250 yard shot at anything is rare.

Straight out of the carton, this rifle delivers fine accuracy as long as the barrel is allowed to cool after three rounds. The light tube is so thin you can almost see the rifling from the outside. With any and all ammunition, five-shot groups run between mediocre and awful.

My standard predator and jackrabbit load is the 85-grain Sierra boat-tail hollowpoint seated in Federal brass. Winchester Large Rifle primers ignite a full charge of IMR-4350 for almost 3,000 fps at the muzzle.

I ran parallel tests with this ammunition, using eight three-shot groups and four groups of five, to prove my point and exercise my newfound

knowledge. The first three shots in each group of five were plotted for inclusion in the three-shot average, as well as to demonstrate how poorly the rifle shows when the last pair of rounds are registered.

The average of four five-shot groups was 1.46 inches. The smallest of the four was .99 inch, with the first three rounds going into .26, and the largest was 2.06 inches. Casting back to the factors used in predicting probable long-run limits from test averages, we see that the upper limit is 1.80 inches, while the lower limit is 1.12 inches. These are not happy numbers when rather small targets are to be taken at 200 yards and more. Indeed, the average of almost 1.5 MOA would not engender absolute confidence in a big game load, especially since the sample size is so meager and the worst case is nearly 2 MOA.

Eight groups of three shots averaged .65 inch. The *largest* was under 1 MOA. Again using the factors of .77 and 1.23, we arrive at probable limits of .50 to .80 inch for the long run average if strings are held to three rounds.

The three-shot groups are both statistically meaningful and pertinent to the system's practical applications. Were I to depend only on the data derived from five-shot groups, this good rifle would need a new owner.

Beyond the firearm and its ammunition, other factors may encourage the use of three-shot groups. The greater the number of shots in a group, the more human error, fatigue and adverse range conditions intrude on the objective quality of the resulting information. Reliable data is a bit more difficult to come by with three-shot groups, but the effort and expense may be justified.

I must stress that three-shot groups cannot be compared with fives or 10s in any statistically meaningful way without undertaking a complex and protracted conversion process. Speer's Dr. Banister described the conversion as "messy," so I don't even want to think about it. Neither can we estimate from three-shot groups what the five- and 10-shot averages would be; at least not at this time.

Nevertheless, now that I know what they mean, I intend to use three-shot groups for assessing accuracy when practical considerations make them appropriate. I thank John Wootters, Dr. Oehler, Messrs. Forker and Thomas, and most especially the gentlemen from Speer for giving me the option.●